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Paula Marasović, Željko Penava

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Modelling the Stress-Strain Curve of Plane-Weave Fabric with Mathematical Models

Paula MARASOVIĆ*, Željko PENAVA

Department of Textile Design and Management, University of Zagreb Faculty of Textile Technology, Prilaz baruna Filipovića 28a, Zagreb, Croatia

*paula.marasovic@ttf.unizg.hr

Article

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ABSTRACT

To avoid fabric deformations due to plastic deformations, it is vital to predict how the fabric will behave under load in advance. This was the starting idea for the conducted research. The basic physical and mechanical properties of plain-weave fabric samples with the same density of warp and weft and in various angles were investigated in this paper. A comparison of the experimental and computational modulus of elasticity was performed. Based on the experimental results, a mathematical model and the corresponding mathematical equations of the stress-strain curve for the examined fabric samples were set up. Finally, a statistical analysis of the models and an evaluation of the parameters and representative indicators of the mathematical models have been implemented. The experimental values of the stress-strain curves are well described by all three of the chosen models. Still, mathematical model number three M3 is best suited and can be used with great accuracy in the theoretical calculation of breaking force and elongation for the selected type of fabric.

KEYWORDS

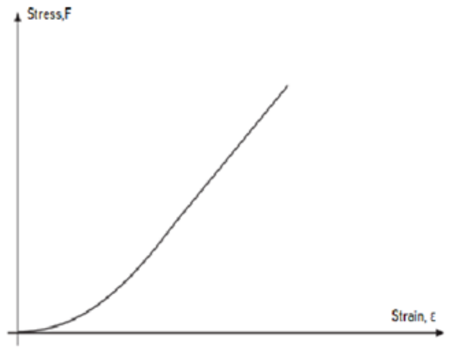
plane-weave fabric, stress-strain curve, anisotropy, the mathematical model, nonlinear regression

INTRODUCTION

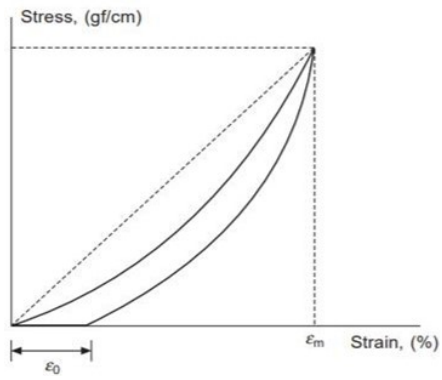
Woven fabrics are generally non-homogeneous, anisotropic, and discontinuous objects. Orthotropic materials are special types of anisotropic materials that have two mutually perpendicular elastic symmetry plains. These elastic symmetry plains are orthotropic plains with cross-sections as orthotropic axes (x and y). As a result, the woven fabric is an orthotropic elastic material. There are two primary directions in a biaxial woven structure: longitudinal (warp, y-axes) and transverse (weft, x-axes). Fabric behaviour is frequently complex; hence experimental verification of theoretical predictions is more crucial for them than for other materials. The "uniaxial test method," or force action in only one direction, is a method for assessing the anisotropic tensile properties of fabrics [1]. The elastic constants change with the angle of action of the external load (tensile force). Although most experimental studies assess deformations and stresses for fabric breakage when a force operates longitudinally or transversely, fabric behaviour and deformation are equally relevant when the angle of action changes, especially in technical fabrics [2]. Because it affects the behaviour of woven fabrics, researchers initially concentrated primarily on the shear behaviour of woven fabrics in both principal directions. Later, their focus shifted to the shear qualities of woven fabrics in other directions [3]. Material simulation and prediction are frequently desired, particularly for materials whose properties are dependent on the primary manufacturing process. Tensile testing is a typical approach for characterizing materials in industry and research. The material's interior behaviour is reflected in the tensile test curve. The curve is like a unique fingerprint showing the relationship between load and elongation. We can have a better understanding if we investigate the responses of tensile and compression loads more thoroughly [4].

Tensile stress-strain curve

One of the essential aspects influencing fabric performance in usage are tensile properties [5]. A textile tensile strength is the highest load it can support before breaking under uniaxial tensile loading. The fabric goes through a loading and unloading cycle to identify the stress-strain curve of woven fabrics [6]. The stress-strain curve of a textile structure subjected to tensile deformation is nonlinear. The nonlinear behaviour of fabric could be explained by the synthesis of straightening of bent yarns and their subsequent elongation. Longitudinal elongation of yarns appears with the increase of the load [7]. Hooke's law expresses the relationship between stress and strain, in which elastic constants are used in the technical literature as engineering elastic constants or apparent elasticity constants of woven fabric [8]. Because each fabric is composed of many different constituent fibres and threads, even minor deformations create a cascade of complex movements [5]. According to research on the behaviour of woven fabrics under tension, small tensile forces result in a sizable membrane strain. The straightening of the fabric's crimped constituent yarns may be the cause of this deformability. The fact that the fibres straighten out under tensile loading or tension at low stresses may help explain this woven fabric behaviour [9]. This complicates matters further because fibres and yarns act non-Hookean under deformation and exhibit hysteresis with time effect. The surface of the hysteresis loop represents the energy spent on continuous deformations during one load cycle [5,10]. Figure 1 shows an example of a woven fabric's tensile stress-strain curve.



a)



b)

Figure 1. a) Tensile stress-strain curve; b) Loading and unloading cycle in the tensile stress-strain curve [5]

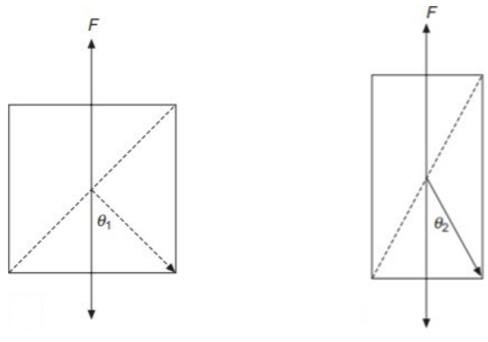
The early portion of this curve has a low slope due to decrimping and crimp-interchange before rising steeply to its height, which can be attributed to induced fibre extension [6]. The level of yarn crimp and the relative ease of yarn deformation also determine the amplitude of the stress-strain curve's summit [5]. If the fabric undergoes a cyclical loading process, it is first stretched from zero stress to a maximum (loading), and then the stress is fully released (unloading) [6]. Due to the viscoelastic nature of textile materials, the residual strain will be observed, ε_0, due to which the recovery curve never returns to the original state, as seen in Figure 1a. Due to hysteresis, a warped fabric cannot return to its former geometrical conditions. In Figure 1b, the shift to the right from the origin of the unloading curve depicts the hysteresis effects extent and the number of permanent sets resulting from the loading history [5].

Extensions in principal and bias direction

By stretching the fabric in any of the main directions, the wrinkled yarn from the weaving process begins to straighten in the direction of the force. Consequently, the interaction between the two yarn thread systems must be taken into account. An increase in the contact between the warp yarn and the weft decreases the amplitude of the yarn and the weaving angle. These yarns appear to become less flattened after tensioning due to their consolidation into a rounder or more circular cross-section, as discovered by Hearle et al. (1969) [11]. In addition, at the crossover point, the crimp level of one pair of yarns will rise while the other drops [4]. The behaviour of the fabric under tensile loads is largely influenced by the crimp in the warp and weft yarns. When a uniaxial tensile load is applied to woven fabric, the crimp decreases in the direction of the load while increasing in the cross direction. This phenomenon is known as crimp imbalance [12]. Within the yarn at the interlacements of two thread systems of the fabric, individual movement of the fibres occurs, which allows the fibre to avoid the stresses that extension can cause. De Jong and Postle (1978) stated that there are six independent dimensionless parameters to examine in the case of uniaxial tensile characteristics of plain woven fabrics in the case of a balanced woven fabric (made from the identical warp and weft yarns). Parameters such as that, e.g. a yarn's rigidity, are defined by several parameters, including the ratio of warp to weft yarn length per crossing yarn and the yarn compression index [9,12]. They also argued that yarn extension might explain how the ratio of yarn compression rigidity to bending rigidity affects relative fabric extension. Yarn extension can account for a significant amount of fabric extension when this ratio is low [5].

One of the challenges in studying woven fabric tensile behaviour is that every extension that occurs at an angle to the warp or weft direction usually includes a different mechanism of deformation [5]. If there is an extension at an angle, we must consider the fabric's shear behaviour. If the fabric is extended at an angle, the trellis members' positions concerning one another will be rotated to accommodate the extension [6]. Fabric properties affect the shear deformation. Due to the yarns' relative motion restriction, the shear resistance as the textiles' areal densities increased [13]. The load applied in the bias direction causes the fabric to shear, which results in the shear angle. In the 45° direction to the warp and weft, for example, the modulus are nearly entirely governed by the fabric's shear behaviour; however, when it is extended in the warp or weft direction, the shear behaviour has no bearing. As a result, a fabric's tensile performance appears to result from a multi-directional effect, called „anisotropy" of woven fabric tensile properties [5].

Fabric tensile behaviour will appear different from when the extension happens just in two primary directions. When a plain-woven fabric is stretched to its final state in a bias direction, as shown in Fig. 2, the warp yarn rotates, bringing the maximum elongation closer to the direction of force (F). At the final location, there is a deviation in the direction (θ1 > θ2). The fabric will be strained to attain its final position if a force is applied. The departure of the angle to the warp direction determines the magnitude of the strain, with the diagonal direction (± 45°) presenting the largest changes in length. As a result, the maximum elongation will occur on one diagonal, whereas the maximum contraction will occur on the other diagonal [5].



a) b)

Figure 2. Fabric extension in bias directions: a) initial position before extension; b) final position after extension [5]

If stress is applied at an angle to the warp or weft direction, the mode of deformation will become a combination of extension and shear. When extended in the bias direction, there will be relatively little tension in the yarns. Still, because of frictional forces at the interlacement points, there will be tension between two adjacent interlacements as well as bending, which deforms the interlaced yarns [9]. The values of tensile properties in the warp direction are considerably higher than those in the weft direction because a stronger crimp is typically generated in the warp yarns [5].

Modelling the tensile behaviour of woven fabric with the mathematical model

Hooke's law derives the basic mathematical model for all types of materials from the elastic model. According to the main principle of this model, the deforming material will demonstrate displacement linearly with the applied force. The well-known equation for this is $F=k*\varepsilon$, where F represents force, k represents slope, and ε represents elongation. The constant and the displacement are the only two necessary parameters [14].

Here are the essential characteristics that a mathematical model should have:

- The model must be capable of capturing the important features of the stress-strain curve under different loading conditions
- The model must be based on reliable theoretical frameworks
- The required parameter must be obtained easily from standard tests so that the applicability of the model will not be hindered by the non-standard experiment that is hard to replicate [14].

Appropriate theoretical equations should be devised to forecast ultimate fabric strength and tensile elongation, which should correlate well with experimental values. Determining normal stresses and strains in various directions requires complicated procedures that offer information on the fabric's physio-mechanical characteristics in various directions. The angles between two systems of threads vary at the places (or sites) where they cross. Fabric samples are tested in the laboratory under the action of tensile force in different directions to determine tensile strength, tensile forces, and tensile elongations due to the inherent nature of textiles. Both the braking forces and the elongation at break can be varied by changing the angle of the external load (tensile forces) [15].

The majority of earlier research into woven fabric tensile behaviour focused on predictive modelling, which always entailed extremely difficult mathematical relationships between stresses and strains. Furthermore, the predictability of these models is not always reliable. Kawabata and Bassett, in particular, used numerical modelling methods to suit the tested tensile curves [16]. Kawabata et al. reported and validated a linearization approach for modelling the biaxial tensile stress-strain relationship of fabrics [5].

To obtain a satisfactory model for the tensile stress-strain relationship of fabric, some principles must be followed:

- The proposed function must be in the correct function group, such as exponential or power.
- It should have a format that is easy to compute or interpret. The non-linear regression approach can rarely adequately evaluate a function with more than four parameters.
- It should satisfy the initial conditions of a physical process, e.g., when force equals zero, strain equals zero.
- Ideally, the parameters of the chosen function will be related to the physical qualities of the yarn and the fabric structure for the desired use [5].

Peirce (1937) was a pioneer in the study of woven fabric tensile deformation. His model assumed that the yarns in the fabrics have a circular cross-section, although this assumption of a circular yarn cross-section in the fabric is purely theoretical. He developed the simplest theoretical model of yarn configuration. However, even with this straightforward model, transcendental functions are still used in the computations for the geometrical parameters [17]. As a result, numerous researchers altered his geometrical model to study tensile behaviour. Grosberg and Kedia (1966) investigated the minor strain within the first load–extension curve using Peirce's rigid thread model [18]. Olofsson (1965), assumed that the cross-yarn would "flow" into the available space and take on the shape of an elastic material being bent by point loads acting at the intersections. He also hypothesized that the yarn would acquire shape caused by such pressures [19]. Leaf (1980) describes three methods for analysing the tensile behaviour of plain-woven materials. For modest deformations only, his initial technique is based on Gastigliano's theorem [20]. For the analysis of massive deformations, a force equilibrium method and an energy approach are used [5].

The elastic modulus is a measure of a material's resistance to elastic deformation and is one of the most important mechanical properties. Finite element analysis (FEA), commonly utilized to model and simplify engineering issues, was used by Tehrami et al. to investigate the tensile behaviour of woven materials with different weave patterns. Lin (2014) sought to predict elastic characteristics to anticipate woven fabric tensile damage behaviour. Chen et al. (2008) examined the tensile behaviour of PVC-coated woven membrane materials when subjected to uniaxial and biaxial loads. Lin et al. (2008) employed a modelling approach to investigate the mechanical behaviour of textile reinforcements; the model represented essential interactions by simulating combinations of compaction and in-plain shear stress [21].

Weissenberg developed the trellis model to define theoretically woven fabric's planar stress-strain relationships. The orientation of the framework of the trellis to the direction of the pull would vary depending on the direction of pull in the material in its natural and applied form. In addition, the directions of maximum elongation and contraction would rotate, bringing the direction of maximum elongation closer to the direction of the pull in the terminal position. He described the material as avoiding the draw by revolving around and having its maximum extension in a direction opposite of the pull [8]. Huang et al. investigated the stress at representative inner nodal sites to see why there was a variation in the equivalent modulus between the single strand, uniaxial, and biaxial tensile tests. The degree of crimp arising from the production process influences the equivalent modulus of the yarn during mechanical loading for the same material with the same initial tensile modulus; the more curled the yarn, the smaller its tensile modulus. At the same crimp, a higher friction force causes the yarns to have higher internal stress, making them more difficult to stretch and resulting in a higher equivalent modulus than during uniaxial tension. Furthermore, the equivalent modulus drops in the following order with the same level of crimp: biaxial test, uniaxial test, single yarn test [21].

Ng S-p et al. concluded that the tension stiffness of all the fabrics decreases as the bias angle increases, ceasing to decrease at 45° or 60° for all fabrics. Based on the modelling data, the corresponding bilinear approximation was created. One can use the trial function to determine the tangent modulus, which can then be utilized to compute the stress value using the suitable piecewise-linear curve of stress-strain relation, to determine the stress at any given bias angle. As a problem, they pointed out that the plain woven fabric is the easiest to model because the weakest tangent moduli are around the true bias of 45°. More complex fabrics may have more than one locally weakest tangent modulus [22]. In the study by Büyükbayrakta, the warp direction produced higher breaking load values while examining the shear deformation characteristics, shear angle, breaking force, and fabric extension at a bias direction. Weft breaking load and bias direction were shown to have a similar relationship. Bias-extension test results for fabric were found to be higher than tensile test results in both warp and weft directions. The direction of applied force influences the load-displacement curve of the fabric, and the setting and weave characteristics of the fabric are significant during tensile and shear deformations [13]. Investigating the biaxial tensile characteristics of stratospheric airship envelope fabrics, the stress-strain curves revealed nonlinear elastic characteristics of envelope fabrics in eleven stress ratios. When the significant load was applied in the warp direction, the maximum warp strain increased marginally as the normalized stress ratios increased. The polynomial technique has shown to be unable to guarantee the accuracy of fitting results in all stress ratios. Still, the correctness of the response surface could be assured by controlling the R-square value to 0.95 or higher, as Gao et al. concluded [23]. Serwatka et al. looked at the stress-strain curve as a signal in time, comparing the stress to the system's dynamic reaction. A new model for the yarn stress-strain curve was proposed, which stands out among the various models reported in the literature. Analysis of sensitivity curves allows seeing how each parameter affects the models and defines their scope. Many of the parameters of the models presented in the literature have a broad scope that spans two or three zones. However, the parameters affect only one zone in their model, which is accurately defined [24]. The force-deformation curve fitting models for chosen woven fabrics, including plain weave, satin, and twill weave, are given in the work by Šomođi et al. Linear, polynomial for powers of 2 and 3, cosine, and sigmoidal curves were chosen as curve fitting models. The used fabric and the chosen model determine the fitting amount. Although cosine and, in particular, sigmoidal curves provide high accuracy, it is worth mentioning that a far simpler parabolic approximation, as well as linear approximation for satin and twill weave, also offer excellent results [25].

This paper aims to compare the experimental and computational modulus of elasticity, i.e. create mathematical models based on experimental results, set the appropriate mathematical equations of the stress-strain curve for the examined plain-woven fabric and, finally, through statistical analysis of the model and evaluation of all statistical parameters, assess the representativeness and correlation of the mathematical model and the curves obtained from the experimental results for the fabric sample.

MATERIALS AND METHODS

In this paper, the basic physical and mechanical properties, i.e. mass per unit area, thickness, fineness, weaving, density and tensile properties of cotton fabric with plain weave produced in the Čateks Ltd. factory from Čakovec were examined.

Mass per unit area was determined according to the ISO 3801:1977 standard, the thickness of the woven samples was determined by the ISO 5084:1996. In contrast, the density of the woven fabric or the number of warp and weft threads per unit was determined according to the HRN EN 1049-2:2003 standard. Weaving or the crimp of yarn in woven samples was determined according to ISO 7211-3:1984, where warp and weft crimp is stated in percent as a length difference between woven yarn and straightened yarn in relation to the length of yarn in the fabric. The fineness of the yarn was determined by the tendril method with 100 m long tendrils. Finally, the Textechno Statimat M tensile tester was used to test the tensile properties of fabric samples in accordance with the standard test method HRN EN ISO 13934-1:2008 [26]. The testing of tensile properties of fabrics was conducted in the directions of warp and weft, and under angles of 15°, 30°,45° and 75°, under a load of 1 N on the lower clamp.

In the following diagram (Fig. 3 and 4), we will observe the curves from 0 to point 1 of the local maximum. The point belonging to the 1st local maximum represents the material's flow or yield limit. After that point, the deformations grow without increasing the load and plastic deformations occur in that area. With a constant load, the fabric pattern lengthens. The obtained curves will be approximated with mathematical expressions in the range up to the 1st local maximum.

The first local maximum selected is the one up to which the curves have a linear (rectilinear) part representing the fabric's elastic part where the thread moves. Then the curve comes to the nonlinear part.

As the force increases, elastoplastic deformation occurs. With a further increase in the force, the yarn begins to deform, and the fabric takes on permanent (plastic) deformations. In practice, a so-called material creep can occur before breaking after a further increase in the force, and such a curve appearance is difficult to model with a mathematical model.

The definition of a local maximum is:

The function ƒ has a local maximum in x_0, if there is an interval <a, b > which contains x_0, so it is valid that $ƒ(x_0) \geq ƒ(x)$, for each $x \in$ <a,b> [27].

The selected mathematical models for modelling stress-strain curves are following:

Mathematical model **M1**: $F = F_{pn} + a\varepsilon^{b}$ (1)

Mathematical model **M2**: $F = F_{pn} + [1 - e^{-bs}]$ (2)

Mathematical model **M3**: $F = F_{pn} + e^{a+bs+cs^{2}}$ (3)

where F_{pn} is prestress (N).

The formulas also contain a, b and c, which are unknown parameters that will be determined by nonlinear regression. Also, these three models must be processed statistically. Nonlinear regression is based on an interaction procedure we will take as the convergence criterion ($1e^{-8}$).

Nonlinear regression is a regression analysis involving fitting data to a model and then expressing the result as a mathematical function. Simple linear regression (y = mx + b) connects two variables (X and Y) with a straight line, whereas nonlinear regression connects the two variables in a curved connection. The model's purpose is to reduce the sum of squares to the smallest possible value. The sum of squares is a metric for determining how far the Y observations differ from the nonlinear (curved) function used to predict F. It is calculated by calculating the difference between the fitted nonlinear function and each Y point in the data set. Nonlinear regression uses logarithmic functions, trigonometric functions, exponential functions, power functions, Lorenz curves, Gaussian functions, and other fitting methods [27].

RESULTS AND DISCUSSION

Testing the material, organizing and analyzing the generated data, and finally fitting a mathematical model are all processes in this research. Therefore, the first step was to test the samples and analyse all the obtained data.

Physical-mechanical properties

The results of testing the physical-mechanical properties are shown in Table 1.

Table 1. Basic parameters of the fabric structure

	Mass per unit area, gm^{-2}	Thickness, mm	Density, thread/cm		Weaving, %		Fineness, tex	
			Warp	Weft	Warp	Weft	Warp	Weft
1	171.80	0.46	50	30	4.76	5.66	24	24
2	169.33	0.45	50	30	5.66	4.76	24.6	24
3	168.31	0.46	50	30	4.76	3.85	24	28
x	169.81	0.46	50	30	5.06	7.76	24.2	25.33
σ	1.79	0.01	0	0	0.52	0.91	0.35	2.31
CV (%)	1.06	1.26	0	0	10.24	19.05	1.43	9.12

According to the results, the tested fabric weight was 169.81 g/m^2, thread density 50 and 30 threads/cm, respectively, with the weaving smaller at the warp than at the weft. The thickness of the fabric was determined to be 0.46 mm, and the fineness of the yarn was 24 tex and 25 tex, respectively. According to the presented results, the parameters of the tested fabric are all relatively uniform and evenly distributed due to the relatively low coefficient of variation and statistical deviation.

Tensile properties

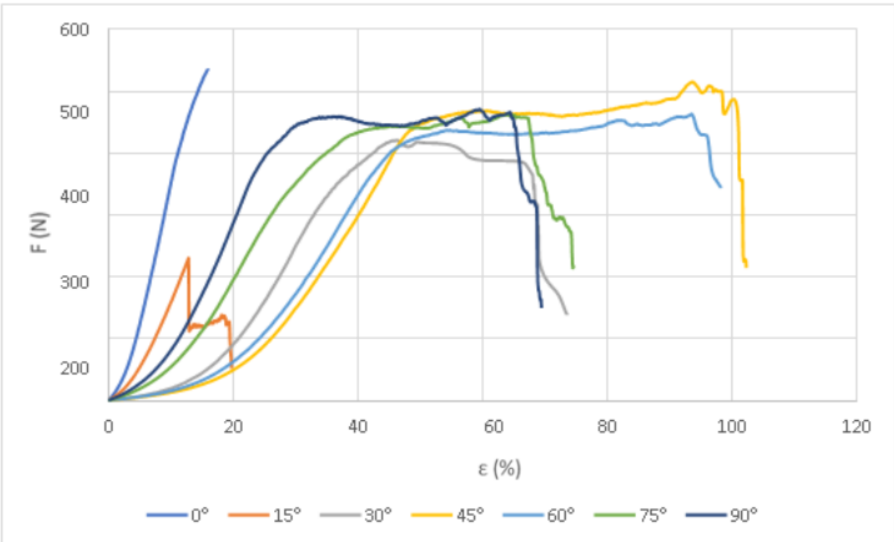


Figure 3. Mean value of tensile force and elongation for plain-woven fabric samples

Figure 3 shows a graph of the mean value of tensile force and elongation for plain-woven fabric samples in each direction. The maximum force of the specimens in the plain weave has the highest value, and the corresponding elongation has the lowest value when the specimens are cut in the weft direction (φ=0°). There is a trend where the angle φ increases, the elongation values also increase, and the values of the maximum force decrease. The exception is the sample at an angle of 15°, with the lowest breaking force of all tested samples. The curves do not start from zero but have a section on the force axis due to bias.

Fitting the stress-strain curves with a mathematical model

After all tested parameters have been defined, a part of the fitting curve with the mathematical model in the Excel program followed.

The next step was to show the stress-strain curves up to the first local maximum where the creep of the material is eliminated and, based on these curves, mathematical models would be approximated. This method greatly reduces the complexity of the experimental dataset and makes it compatible with various mathematical models that reduce biaxial behaviour to a set of constants.

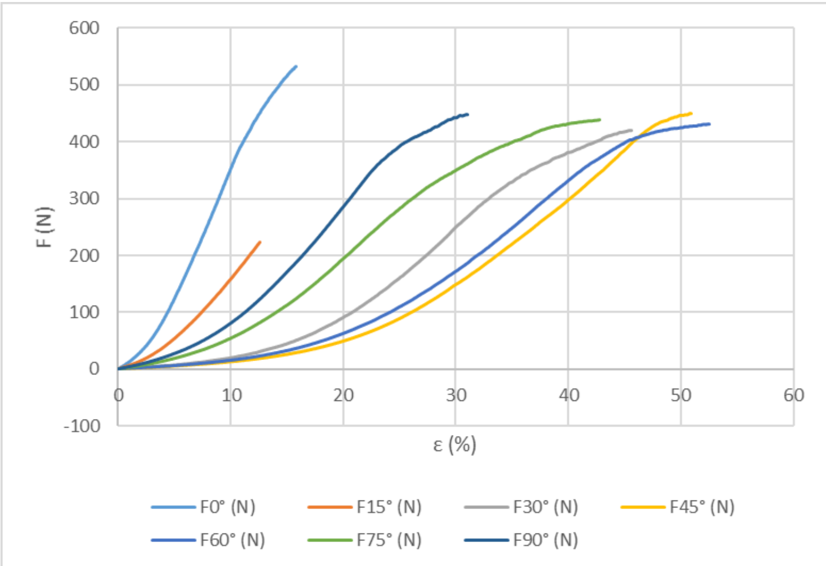


Figure 4. Mean value of tensile force and elongation to 1st local extreme (maximum) for fabric samples

The next step is calculating the curves' values for all the angles examined.

Table 2. Mean value of the prestress force, correlation coefficients between the line and the curve on the linear part, maximum elongation, force and work, elongation at break, breaking force, work at break, elongation, force and work at the 1st local maximum for plain-woven fabric samples

	0°	15°	30°	45°	60°	75°	90°
Fps (N)	1.014	1.008	1.059	1.084	1.062	1.050	1.001
YSlope	12.485	6.550	1.212	0.966	1.154	2.911	4.187
YInter (N)	1.014	1.008	1.059	1.084	1.062	1.050	1.001
YCorre	0.9977	0.9995	0.9972	0.9992	0.9986	0.9993	0.9999
εmax (%)	15.96	12.80	46.20	93.72	93.52	63.84	59.52
Fmax (N)	533.1	228.9	419.7	513.5	461.8	460.8	469.7

	0°	15°	30°	45°	60°	75°	90°
W_{max} (N*mm)	8178.4	2351.2	14882.9	24132.7	23384.7	22286.8	15655.3
ε_{break} (%)	16.04	19.72	73.60	102.32	98.28	74.60	69.44
F_{break} (N)	533.0	50.8	137.6	213.4	342.7	213.5	148.4
W_{break} (N*mm)	8372.1	4035.4	35562.8	63460.8	57844.3	45432.6	46349.2
ε_{1LM} (%)	15.84	12.60	45.60	50.88	52.52	42.80	31.00
F_{1LM} (N)	532.6	223.7	418.8	449.1	430.6	439.3	447.6
W_{1LM} (N*mm)	7882.9	2137.6	15149.8	15120.6	17801.5	18512.9	12442.2

F_{ps} is the prestress force, Y_{Slope}, Y_{Inter} and Y_{Corre} are correlation coefficients between the line and the curve on the linear part of the curve in which Hooke's law applies, ε_{max} is the maximum elongation, F_{max} is the maximum force, W_{max} is the maximum work, ε_{break} is the elongation at break, F_{break} is the breaking force, W_{break} is the work at break, ε_{1LM} is the elongation at the 1st local maximum, F_{1LM} is the force at the 1st local maximum, W_{1LM} is the work at the 1st local maximum.

The following is the definition of unknown variables for fitting a mathematical model, as shown in the following table.

Table 3. Coefficients a, b and c of plain-woven fabric samples for each mathematical model

φ		0°	15°	30°	45°	60°	75°	90°
M1	a	19.36237	4.6167	0.54474	0.06801	0.30797	4.59821	3.64746
	b	1.22972	1.53425	1.77245	2.26311	1.8657	1.24776	1.43429
M2	a	-741.641	-77.2663	-100.423	-39.5824	-85.967	-631.193	-263.249
	b	-0.03646	-0.11007	-0.03868	-0.05189	-0.03693	-0.01358	-0.03469
M3	a	3.03236	2.01663	1.04904	0.80462	0.87092	2.56386	2.25399
	b	0.41929	0.45976	0.2244	0.18885	0.2012	0.17865	0.25538
	c	-0.01373	-0.01529	-0.00254	-0.00166	-0.00195	-0.00228	-0.00426

Unknown parameters a, b and c were calculated for each mathematical model, then the standard error, the correlation coefficient of the mathematical model and Anova statistical test of the mean value of the square error was performed, which showed the following results.

Table 4. Standard error of the coefficients of plain-woven fabric samples for each mathematical model

φ		0°	15°	30°	45°	60°	75°	90°
M1	a	0.47102	0.01774	0.02339	0.00178	0.01297	0.14702	0.11438
	b	0.00976	0.00167	0.0119	0.00698	0.01121	0.00918	0.00984
M2	a	54.25364	1.52296	3.5022	0.82083	2.65426	39.89356	11.27608
	b	0.00215	0.0013	0.000709	0.000426	0.000565	0.00069	0.000996
M3	a	0.02196	0.0218	0.01162	0.00872	0.00446	0.01475	0.00899
	b	0.00405	0.00493	0.000686	0.000447	0.000226	0.000997	0.000818
	c	0.000182	0.000272	$9.95E^{-06}$	$5.65E^{-06}$	$2.82E^{-06}$	$1.64E^{-05}$	$1.82E^{-05}$

Table 5. Correlation coefficients of plain-woven fabric samples for each mathematical model

Math.model	0°	15°	30°	45°	60°	75°	90°
M1	0.99021	0.99986	0.98213	0.99587	0.9836	0.97807	0.98633
M2	0.98469	0.99719	0.96617	0.98506	0.96792	0.97013	0.97643
M3	0.99642	0.99793	0.99928	0.99972	0.9999	0.99648	0.99933

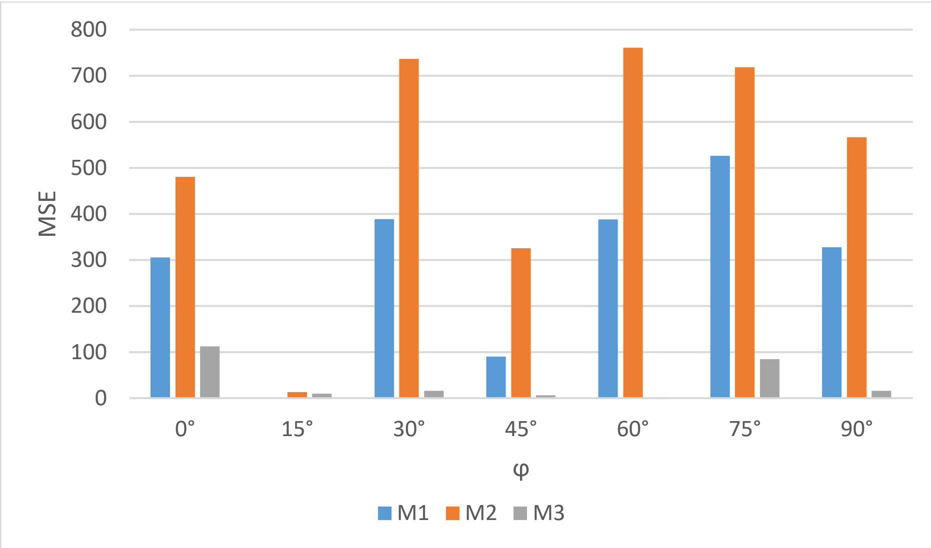
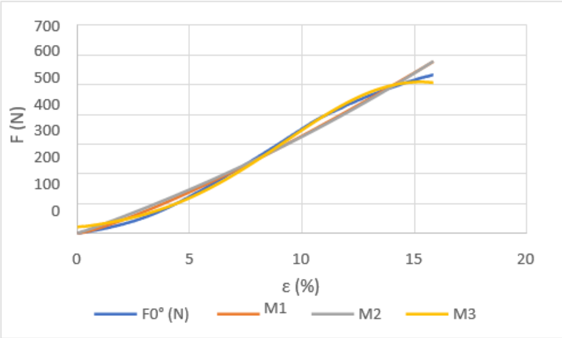


Figure 5. ANOVA Mean Square Residual - mean squared error (MSE) of PL fabric sample for eachmathematical model

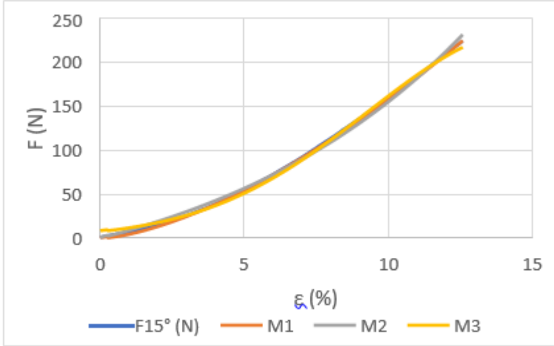
From the obtained results it can be seen that all three selected models approximate the experimental curves well. The standard error in all samples is the highest in the mathematical model M2 where the values are incredibly high in the unknown parameter a. The smallest standard error is in the mathematical model M3, where the error values were almost 0. Accordingly, the correlation coefficient of theoretical curves is obtained by the mathematical model, and experimental curves at all angles are best fitted with the mathematical model M3, i.e. the highest correlation coefficient is observed in model 3. Therefore, it can be concluded that mathematical model 3 (M3), given the low error values and high correlation coefficient values, best describes the experimental values of the force-elongation curves.

Graphs from experimentally obtained curves and curves for three mathematical models for various directions of force action are shown in Figure 6.

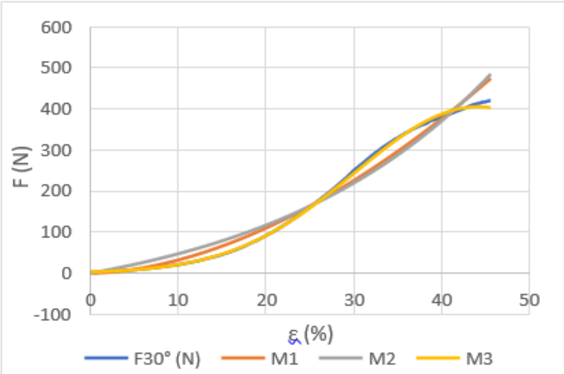
https://doi.org/10.31881/TLR.2022.33



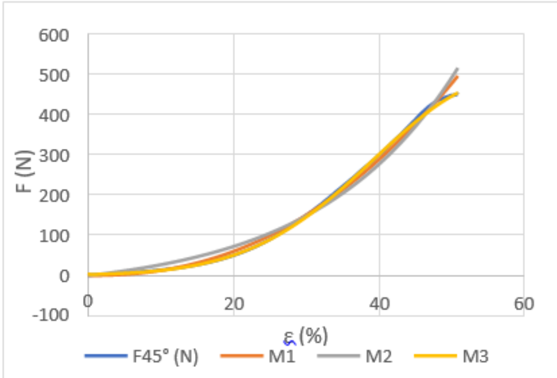
a)



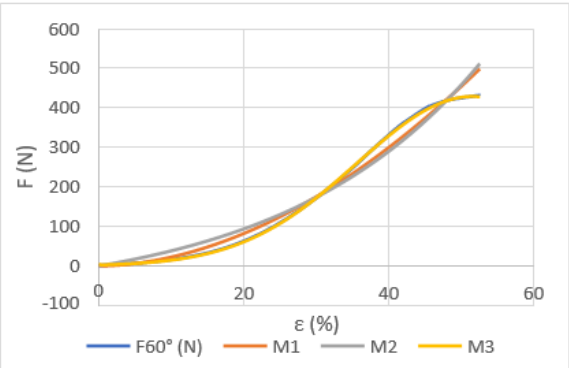
b)



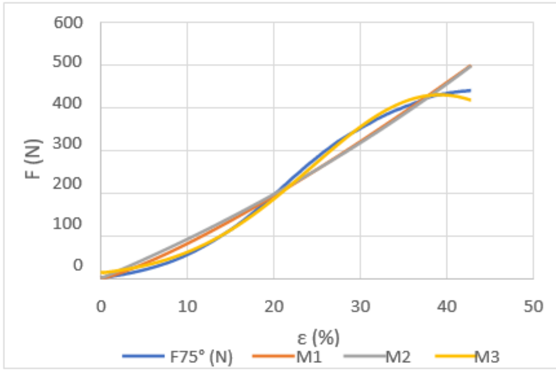
c)



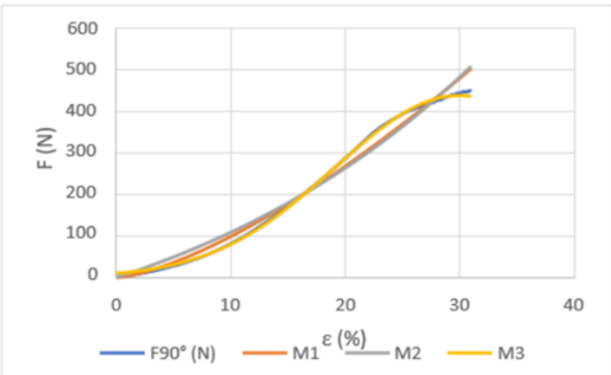
d)



e)



f)



g)

Figure 6. Stress-strain curves of experimental curves fitted with three mathematical models for plain-woven fabric sample: a) for 0°, b) for 15°,c) for 30°, d) for 45°, e) for 60°, f) for 75°, g) for 90°.

From the obtained graphs, it can be seen that all three mathematical models describe the experimental curves well, but mathematical model three (M3) has the highest degree of correlation, the lowest standard error, and the lowest mean square error of all of the models. Thus, it can be concluded that for the plain-woven fabric, according to the performed statistical tests, the mathematical model - M3 - best describes or it is best fitted to the experimental stress-strain curves, especially for fabric samples in the angles 15°, 30°, 45°, and 60°.

CONCLUSION

By examining the samples, it can be concluded that the maximum force of the plain-weave specimens has the highest value in the weft direction and the lowest elongation at break. The elongation increases by increasing the angle while the maximum force values decrease. Three mathematical models were chosen to find the unknown parameters, with nonlinear regression used. These three models were also statistically processed. The resulting correlation coefficient, standard error, and mean square error (MSE) data demonstrate the usefulness of theoretical equations for determining the breaking force and elongation at break for samples of plain-woven fabrics with various angles. All three selected models describe the experimental curves well, but mathematical model three, M3, given the low error values and high values of the correlation coefficient, is best fitted for the experimental values of the stress-strain curves and this model can be used with great accuracy in the theoretical calculation of breaking force and elongation for plain-weave fabric.

Author Contributions

Conceptualization – Penava Ž; methodology – Marasović P and Penava Ž; formal analysis – Marasović P and Penava Ž; investigation – Marasović P and Penava Ž; writing-original draft preparation – Marasović P; writing-review and editing – Marasović P and Penava Ž; supervision – Penava Ž. All authors have read and agreed to the published version of the manuscript.

Conflicts of Interest

The authors declare no conflict of interest.

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